Exam Questions (Algebraic Number Theory)

Everyone gets one of the following topics at the exam randomly. The exam lasts approx. 20 minutes. **There is no preparation time but you may use your own notes (or mine) during the exam.** It is important that you understand everything (including the proofs), you should be able to give examples/counterexamples, should know which lemma depends on what and how we use them to prove the theorems. Beware that the topics may not form a connected subset of the lecture notes.

1. Integral elements in ring extensions. Preliminary properties of trace and norm. Discriminant, existence of integral basis.
2. Dedekind domains, unique factorization of ideals. Localization, discrete valuation rings.
3. Lattices and Minkowski theory, $K_{\mathbb{R}}$ as a Minkowski-space. Estimate of the class number.
4. Extensions of Dedekind domains, fundamental equation for the decomposition of prime ideals. Action of the Galois group on the primes.
5. Ramifying primes and the discriminant. Algorythm for decomposing a prime in a finite extension (except for finitely many). Integral basis and ramifying primes in cyclotomic fields.
6. Valuations (Ostrowski's theorem), completion, the field of p-adic numbers. Hensel's lemma, extending valuations.
7. Classification of local fields. Ramification subgroups and the relation to the multiplicative groups.
8. The proof of the local Kronecker–Weber theorem – the unramified and tamely ramified part.
9. The proof of the local Kronecker–Weber theorem – the wildly ramified part.
10. Direct limit, inverse limit (definition and basic properties). Global fields, the proof of the global Kronecker–Weber theorem.