Exam questions (Algebraic Number Theory)

1. Integral elements in ring extensions. Dedekind's lemma and Hilbert Theorem 90 in Galois theory.
2. Discriminant, existence of integral basis in the ring of integers of number fields.
3. Dedekind domains and unique factorization of prime ideals.
4. Minkowski's theorem in convex geometry. $K_{\mathbb{R}}$ as a Minkowski space.
5. The finiteness of the class number (using Minkowski theory).
6. Dedekind domains and localization.
7. Extensions of Dedekind domains, fundamental equation for the decomposition of prime ideals.
8. Hilbert's ramification theory for Galois extensions of number fields.
9. Absolute values, Ostrowski's theorem, completion, definition of p-adic numbers.
10. Direct and projective limits, exactness properties.
11. Hensel's Lemma and the extension of valuations.
12. Classification of local fields.
13. Ramification subgroups and their relationship with multiplicative groups.
14. Proof of local Kronecker–Weber Theorem – the unramified and the tamely ramified part.
15. Proof of local Kronecker–Weber Theorem – the wildly ramified part.
16. Global fields, proof of global Kronecker–Weber Theorem.